DISCRETE MATHEMATICS PROBLEMS

Lesson 1. Logic

1. Represent with logic symbols the following propositions:

   a) I have nothing.

   b) Yesterday we went to the cinema and we didn't study.

   c) If we are not in a hurry, we will be saved.

   d) I won't buy it, if you don't like it.

   e) I phoned you on Tuesday, but you weren't at home.

   f) Either you print or you make photocopies of the work this morning, or we won't deliver on time and we will fail.

   g) Neither Juan nor Pepe will come tonight.

   h) If the road has building work, then we will take a secondary road, not the motorway.

   i) Animals, like plants, are living beings.

   j) If you have lost your money then, if you don't achieve to recover the data, all the work done up to now will be not useful anymore.

   k) Propositions P and Q are equivalents if and only if the expression $P \leftrightarrow Q$ is a tautology.

2. Show which ones of the following expressions have logical sense and which ones not. Try to represent the correct ones using as less parentheses as you could.

   a) $p \vee (p \wedge q) \wedge s$

   b) $(p \vee \neg q) \to r \vee p$

   c) $p \to \neg r \to s \wedge p$

   d) $((\neg q \wedge p) \to (s \vee p)) \to q$

   e) $[(((p \vee q) \vee r) \to p) \wedge \neg(p \vee (r \wedge \neg q))] \to r$

   f) $[(p \to q) \wedge (r \to q)] \leftrightarrow (p \vee r) \to q$

3. Make the truth table of the following formulas, showing if they are tautologies, contradictions or contingences.

   a) $\neg P \wedge \neg Q$

   b) $P \wedge Q \to R$

   c) $\neg(P \to Q) \wedge \neg(Q \to P)$

   d) $P \to (Q \to R)$

   e) $\neg P \wedge Q \to \neg Q \wedge R$

   f) $(P \to Q) \wedge (R \to Q) \leftrightarrow (P \vee R) \to Q$

   g) $\tau \to P$

   h) $\tau \to \phi$

   i) $\phi \to \tau$

   j) $P \wedge Q \to \phi \vee \tau$

4. Simplify the following propositional forms:

   a) $(\neg P \wedge Q) \wedge P$

   b) $P \to (Q \to P)$

   c) $\neg Q \wedge R \leftrightarrow Q$

   d) $(\neg P \to \neg(\neg P \vee Q)) \to \neg P$

   e) $(P \to (\neg Q \vee R)) \wedge (\neg P \vee \neg R)$

5. A system of hypothesis is said to be **inconsistent** when a contradiction can be deduced from it. Say whether the following system of hypothesis is inconsistent or not:

$$\text{P1: } \neg P \to Q \quad \text{P2: } R \to Q \quad \text{P3: } P \to R \quad \text{P4: } \neg Q$$

6. Prove that the following implication is a tautology using the logic inference methods:

$$(\neg P \vee \neg Q) \wedge (R \to Q) \wedge (S \to P) \Longrightarrow \neg(R \wedge S)$$

7. Prove that the conlusion can be deduced from the hypothesis in each one of the following examples:

a) **P1:** $P \to Q$
P2: $Q \to (\neg S \to R)$
P3: $P \wedge \neg R$
C: S

b) **P1:** $\neg P \to Q$
P2: $R \to \neg Q$
P3: $R \vee S$
P4: $\neg P$
C: $Q \wedge S$

c) **P1:** $\neg R \to Q$
P2: $T \to \neg Q$
P3: $\neg S \to \neg Q$
C: $T \vee \neg S \to R$

d) **P1:** $\neg(\neg P \vee Q)$
P2: $\neg T \to \neg S$
P2: $P \wedge \neg Q \to S$
P4: $\neg T \vee R$
C: R

e) **P1:** $R \to (S \to Q)$
P2: $\neg P \vee R$
P3: S
C: $P \to Q$

f) **P1:** $R \to T$
P2: $T \to \neg S$
P3: $(R \to \neg S) \to Q$
C: $P \to P \wedge Q$

8. Prove that the conlusion can be deduced from the hypothesis in each one of the following examples:

P1: $\neg P \to Q \vee R$
P2: $\neg Q \to \neg P \wedge \neg S$
P2: $S \to \neg Q \vee r$
P4: $\neg P \wedge \neg R$
C: $\neg S$

Are all the hypothesis necessary? Justify your answer.

9. Write the following statements with symbols and deduce logically the corresponding conclusion:

a) "If we do not go to Versailles, we will visit Venice. If we go to Paris, we will not visit Venice. If we do not go to Italy, then we will not visitVenice. Tehrefore, if we go to Paris or we do not go to Italy, we will visit Versailles".

b) "We will go to the theatre or to the outdoor cinema. If it rains, we will not go to the outdoor cinema. We do not go to the theatre or it rains. So that, we will not go to the theatre if and only if we go to the outdoor cinema".

c) "If a ship has run aground and it crashes, there will be a loss of fuel. If there is a loss of fuel and it can not be collected, then lots of fishes will die. If they do not send help, the ship will crash and the fuel can not be collected. The ship has run aground and they have not send help. Therefore, lots of fishes will die".

10. A politician put forward the following:

    "If inflation is not decreasing or the euro is not depreciated, taxes will not rise. If taxes rise, inflation will decrease if and only if the euro is depreciated. Either taxes rise, or the euro is depreciated and inflation decreases. Therefore, taxes rise, the euro will not be depreciate, and inflation will fall.

    Say if the politician's conclusion is logically right or not.

11. Se tiene el siguiente enunciado:

    "If Valencia CF does not win the league, Barcelona supporters will celebrate it, and Valencia CF coach will not have not renewed his contract. If Valencia FC wins the league, Valencia will hold a big party. Barcelona supporters will not celebrate it."

    Prove the following conclusion in two ways, directly and by contradiction:

    "Valencia will hold a big party."

12. Using the specific rules of predicate calculus and the general properties of propositional logic, prove the following equivalences.

    a) $\neg\forall x(P(x) \vee Q(x)) \equiv \exists x(\neg P(x) \wedge \neg Q(x))$
    b) $\neg\exists x(P(x) \vee Q(x)) \equiv \forall x(\neg P(x) \wedge \neg Q(x))$

13. Using the specific rules of predicate calculus and the general properties of propositional logic, prove the following implications. Give an example which shows that the converse implications are false.

    a) $\exists x(P(x) \wedge Q(x)) \Rightarrow (\exists x P(x)) \wedge (\exists x Q(x))$
    b) $(\forall x P(x)) \vee (\forall x Q(x)) \Rightarrow \forall x(P(x) \vee Q(x))$

14. In each case, prove that the conclusion is right:

    a) **P1:** $\forall x\ P(x) \to \neg Q(x)$
    P2: $\forall x\ P(x) \vee R(x)$
    P3: $\forall x\ Q(x)$
    C: $\exists x\ P(x)$

    b) **P1:** $\forall x\ P(x) \to Q(x) \vee R(x)$
    P2: $\forall x\ P(x) \wedge S(x)$
    P3: $\neg Q(a) \wedge S(a)$
    C: $P(a) \to R(a)$

15. Represent formally the following arguments and deduce logically the conclusion:

    a) "Some dogs are of seter breed. All seter breed dogs can hunt. Therefore, some dogs can hunt".

b) "Some sportsmen practice swimming. No sportsman smokes. Therefore, some swimmers do not smoke".

c) "Everyone who goes to the beach wants to get tanned. No people who want to get tanned take a sunbath. Peter does not take a sunbath. Therefore, there are people who do not go to the beach"

d) "All scammers try to defraud others. All who are not scammers are bothered by cheating. Eva never tries to defraud others. Eva is bothered by cheating"

e) "No spaniard is blonde. All nordics are blonde. Therefore, no spaniard is nordic".

f) "No interesting poem is not popular among people of good taste. No old poem has feeling. All your poems are about soap bubbles. No poem without feeling is popular among people of good taste. No modern poem is about soap bubbles. Therefore, none of your poems is interesting".

g) "All crime suspects hate the victim or work with her. None of the crime suspects who work with her knew that she was going to be promoted. The chief's daughther, who was a crime suspect, knew that the victim knew was going to be promoted. Therefore, the chief's daughter hates the victim".

h) "All Discrete Mathematics students know logic. All Discrete Mathematics students which pass the subject know Boole algebras. Rosana is a student of Discrete Mathematics who does not know Boole Algebras. Inés know Boole algebras, but she does not know logic. Therefore, Rosana does not pass Discrete Mathematics and Inés is not a student of Discrete Mathematics".

i) "In an assembly of neighbors, all people over forty years are men without having a driving license, or are women. All neighbours who attend to the assembly are more than forty years and have a driving license. Therefore, there are only women in the assembly".

16. Prove if the following argument is right:

    Coloured flowers are always worthy of admiration. I like the flowers which do not grow on the outside. No flower grown on the outside have no color. So, I do not like any flower which is not worthy of admiration'

17. Justiffy the following deduction:

    "All the desserts are good. This dish is a dessert. No good thing is distasteful. Therefore, this dish is not distateful".

18. Prove the following statements by induction:

    a) $2n + 1 \leq 2^n$, for every natural number $n \geq 3$.

    b) $2^n \geq n^2$, for every natural number $n \geq 4$.

    c) 7^n is an odd number, for every natural number n.

    d) $11^n - 1$ is a multiple of 5, for every natural number n.

    e) $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots n(n+2) = \frac{n(n+1)(2n+7)}{6}$, for every natural number n.

    f) $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots \frac{1}{n(n+1)} = \frac{n}{n+1}$, for every natural number n.

19. Consider the following process:

Paso 1: Let $S = 1$
Paso 2: Print S
Paso 3: Replace S by $S + 2\sqrt{S} + 1$

a) Write the first four printed values of S

b) Prove that the value of S is always a natural number