DISCRETE MATHEMATICS PROBLEMS

Lesson 3. Boolean Algebras and Lattices

1. In a Boolean algebra (or lattice), simplify the following expressions pointing out which properties you are using:

   a) $\overline{(ab + \bar{b} + \bar{a})}$

   b) $\overline{\left(a\bar{b}(a + \bar{a}b)\right)}$

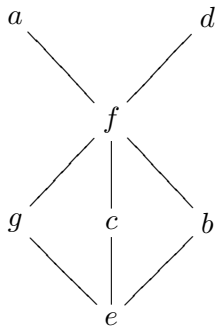
2. In the Boolean algebra $A = \{0, 1, a, b\}$, solve the following equations:

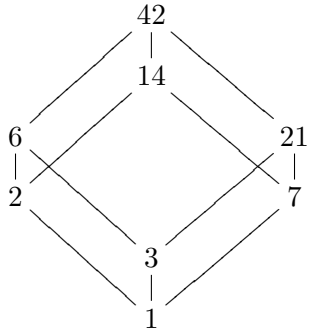
   a) $ax^2 = 0$,

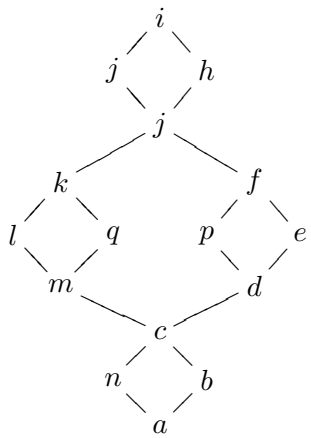
   b) $(a + x)^2(b + x) + x^2 = 1$,

   where x^2 denotes xx.

3. Analyze if the following ordered sets, whose Hasse diagram is shown below, are Boolean lattices:







4. Consider the set D_{36} of the positive divisors of 36 with the divisibility relation. Draw the corresponding Hasse diagram. Give the maximum and the minimum of this lattice. Say which elements have a complement. Is it a boolean lattice?

5. In a Boolean algebra $(A, +, \cdot)$, find the canonical forms of a 3-variable function which takes the value 1 if xyz is the binary expression of a prime number and takes the value 0 if xyz is the binary expression of a composed number (not prime).

6. An assembly of 36 people is called to vote to accept or reject several proposals. The assembly is divided in four groups X, Y, Z, T, which have 5, 8, 10, and 13 members respectively. To each proposal, all the members of the group vote in the same sense and no group abstains. The proposals are accepted if, and only if, they reach absolute majority. Determine the canonical forms of the function $f(x, y, z, t)$ which take the value 1 when the proposal is accepted by an absolute majority (at least 19 votes) and the value 0 in the other case.

7. Simplify the following Boolean function using Quine-McCluskey method:

$$f(x, y, z) = xyz + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$