DISCRETE MATHEMATICS PROBLEMS

Lesson 4. Combinatorics

1. The addition rule

   a) A square with side length equal to 4 is divided by parallel lines into 16 equal squares. What is the total number of squares in the picture?

   b) What is the total numer of movements that a knight can make on an 8×8 chessboard?

2. The multiplication rule

   a) In how many ways can one choose one white and one black square on a 8×8 chessboard? In how many ways can this be done if the squares are not to lie in the same row or in the same column?

   b) In a fruit basket there are 12 apples and 10 pears. John chooses an apple or a pear, and then Jane chooses one apple and one pear. After which of John's choices does Jane have a greater number of choices?

   c) In how many ways can we choose from a complete set of dominoes (28 pieces) two that will fit together (that is the number of dots on one side of the first piece agrees with the number of dots on a side of the second piece)?

3. The inclusion-exclusion principle

   a) Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by $2, 3$, or 5.

   b) In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns *car,dog,ten* or *byte* occurs?

4. The pigeon principle

   a) Given 13 students, then there are at least two of them that have the birthday the same day.

   b) Suppose that you have file with 500.000 "words" of four or less lowercase letters. Can it be that the 500.000 words are all distinct?

   c) Any subset of size 6 from the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ must contain two elements whose sum is 10.

   d) Prove that if 101 integers are selected from the set $S = \{1, 2, 3, \ldots, 200\}$ then there are two integers such that one divides the other.

5. Permutations

   a) How many four-digit positive integers can be written with the digits 1, 2, 3, and 4, such that none of them are repeated? And with the digits 0,1,2, and 3?

   b) Five persons, A, B, C, D, and E are to speak at a meeting, each one of them exactly once. Find the number of all possible orders of their appearance. Which is the number if B is to speak inmediately after A? Which is the number if B does not speak before A?

6. Variations

   a) Seven students were elected as the executive commitee of a student society. In how many ways can a president, vice-president, secretary, and treasurer be chosen from among them?

   b) How many 4-digit positive integers with distinct digits can be constructed with $0, 1, 2, 3, 4, 5$? How many of them are even?

7. Combinations

   a) From among 7 boys and 4 girls we want to choose a six-member volleyball team that has at least two girls. In how many ways can this be done?

   b) At a dance there are 12 boys and 15 girls. In how many ways can we take 4 boys and 4 girls to form four couples consisting of of a boy and a girl?

8. Permutations with repetitions

   a) In how many ways can the following white chess figures be distributed on the first row of a chessboard: 2 knights, 2 bishops, 2 rooks, the king, and the queen.

   b) An anagram of a word is an ordered collection of letters of this word, where each letter occurs as often as in the given word. How many anagrams are there of the word *Bananarama*?

9. Variations with repetitions

   a) The Spanish soccer pool (quiniela) is based on the outcome of 14 games each week. Find the number of possible bets.

   b) Find the number of all subsets of a finite n-element set A.

   c) Find the number of all five-digit positive integers written with the digits $0, 1, 2, 3, 4, 5, 6$, if we allow repetitions of digits.

10. Combinations with repetitions

    a) It is Christmas time! We go to a store where 12 different Chirstmas postcards. In how many ways can a tourist purchase 8 postcards in ordert to send each one to a different address (that is, multiples copies of a postcard can be purchased)?

    b) In how many ways can one distribute 12 coins of one euro over 7 numbered envelopes such that none of them remains empty?